PRESENEČENJA V FIZIKI: VRTAVKE

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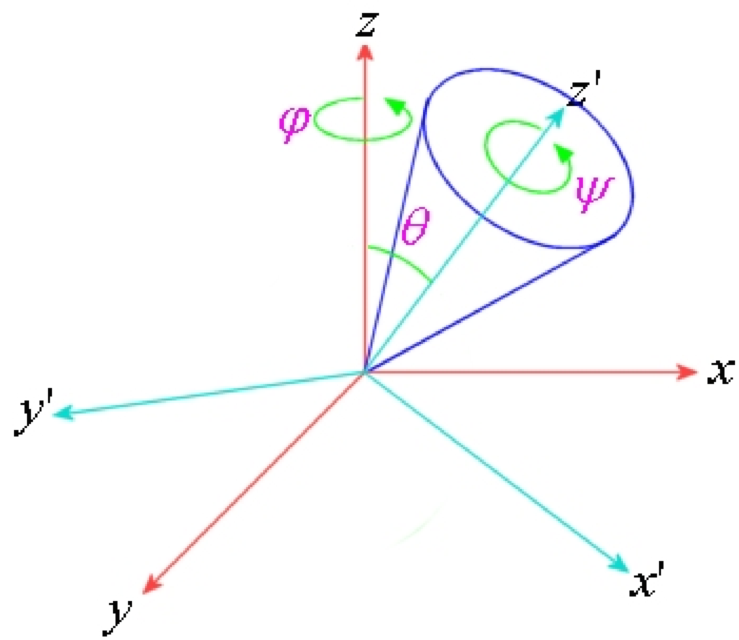
1. Vrtavka na prostem



$$\left(\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}\right)_{\boldsymbol{\tau}} + \boldsymbol{\omega} \times \mathbf{L} = \boldsymbol{\tau}$$

$$\mathbf{L} = \underline{\underline{\mathbf{I}}} \cdot \boldsymbol{\omega}$$

$$\mathrm{L} = \underline{\underline{\mathbf{I}}} \cdot \left(\omega_1 \mathbf{e}_1^{\mathrm{P}} + \omega_2 \mathbf{e}_2^{\mathrm{P}} + \omega_3 \mathbf{e}_3^{\mathrm{P}}\right) = I_1\omega_1 \mathbf{e}_1^{\mathrm{P}} + I_2\omega_2 \mathbf{e}_2^{\mathrm{P}} + I_3\omega_3 \mathbf{e}_3^{\mathrm{P}}$$

Simetrična vrtavka ($I_1 = I_2$)

$$I_1\dot{\omega}_1 + (I_3 - I_1)\omega_2\omega_3 = 0$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1 = 0$$
$$I_3\dot{\omega}_3 = 0$$

$\omega_3 = const$

ω_1 **in** ω_2 **nihata sinusno...precesija!**

$$\begin{aligned} I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 &= \tau_1 \\ I_2\dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1 &= \tau_2 \\ I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 &= \tau_3 \end{aligned}$$

Vrtavka na prostem - stabilnost

Predpostavimo majhna nihanja okrog telesne osi 1, okrog katere se vrti vrtavka, ter $I_1 < I_2 < I_3$ in $\lambda,\ \mu \ll \omega_1$

$$I_1 \dot{\omega}_1 = 0$$

$$I_2 \dot{\lambda} + (I_1 - I_3)\omega_1 \mu = 0$$

$$I_3 \dot{\mu} + (I_2 - I_1)\omega_1 \lambda = 0$$

$$\begin{aligned} I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 &= \tau_1 \\ I_2\dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1 &= \tau_2 \\ I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 &= \tau_3 \end{aligned}$$

$$\dot{\lambda} + \frac{I_1 - I_3}{I_2}\omega_1 \mu \qquad \dot{\mu} + \frac{I_2 - I_1}{I_3}\omega_1 \lambda \qquad \Omega_1 = \sqrt{\frac{(I_1 - I_3)(I_1 - I_2)}{I_2 I_3}}\omega_1$$

Frekvenca je realna! Za vrtenje okrog srednje osi pa dobimo eksponentni pobeg:

$$\lambda(t) = Ae^{\alpha_2 t} + Be^{-\alpha_2 t} \qquad \alpha_2 = \sqrt{\frac{(I_2 - I_1)(I_3 - I_2)}{I_1 I_3}}\omega_2$$

2. Vrtavka na mizi: vrtenje, precesija, nutacija

$$\vec{\omega}_p \times \vec{L} = \vec{M} = \vec{r} \times m\vec{g}$$

Ker sta kota na obeh straneh ista, $\vec{\omega}_p || \vec{g}$, $\vec{L} || \vec{r}$,

smemo znak vektorskega produkta izpustiti:

$$\omega_p \, L = r \, m \, g$$

$$\omega_p = \frac{r \, m \, g}{I_3 \, \omega}$$

3. Vrtavka na mizi: trenje, ustavljanje

R = radij kolesa (vrtavke) a = radij konice (vrtišča)
k = koeficient trenja
Ocenil sem začetno frekvenco 10 Hz in izmeril čas ustavljanje 60 sekund.

$$I = mR^2/2\,,$$

$$\dot{L} = mR^2\dot{\omega}/2 = kg\int r\mathrm{d}m = kg\int_0^a \frac{rm}{\pi a^2}\,2\pi r\mathrm{d}r = \frac{2}{3}kgam\,,$$

$$a = \frac{3}{4}\,\frac{\omega_0/t_0}{k\cdot g}$$

ZGLED:
$\nu_0 = 10\mathrm{s}^{-1}$, $\omega_0 = 2\pi\nu_0 = 60\mathrm{s}^{-1}$, $t_0 = 60\mathrm{s}$, $k = 0,1$, $R = 5\mathrm{cm}$.

$$a = \frac{0.75\cdot 25\mathrm{cm}^2\cdot 60\mathrm{s}^{-1}}{60\mathrm{s}\cdot 0,1\cdot 981\mathrm{cm\,s}^{-2}} = 0.2\mathrm{cm}.$$

4. Ohranitev vrtilne količine

$$L_{\mathrm{prej}} = L_{\mathrm{potem}} \Longrightarrow I_1\,\omega_1 = I_2\,\omega_2$$

$$mR_1^2\,\omega_1 = mR_2^2\,\omega_2$$

Če vztrajnostni radij R skrčimo, $R_2 < R_1$, se kotna hitrost poveča, $\omega_2 > \omega_1$ (pirueta!).

Če je $\omega_2 > \omega_1$, je rotacijska kinetična energija $I_1\,\omega_1^2/2 > I_2\,\omega_2^2/2$ in moramo opraviti delo!

5. Sprememba vztrajnostnega momenta

$$L_{\mathrm{prej}} = L_{\mathrm{potem}} \Longrightarrow I_1\,\omega_1 = I_2\,\omega_2$$

$$mR_1^2\,\omega_1 = mR_2^2\,\omega_2$$

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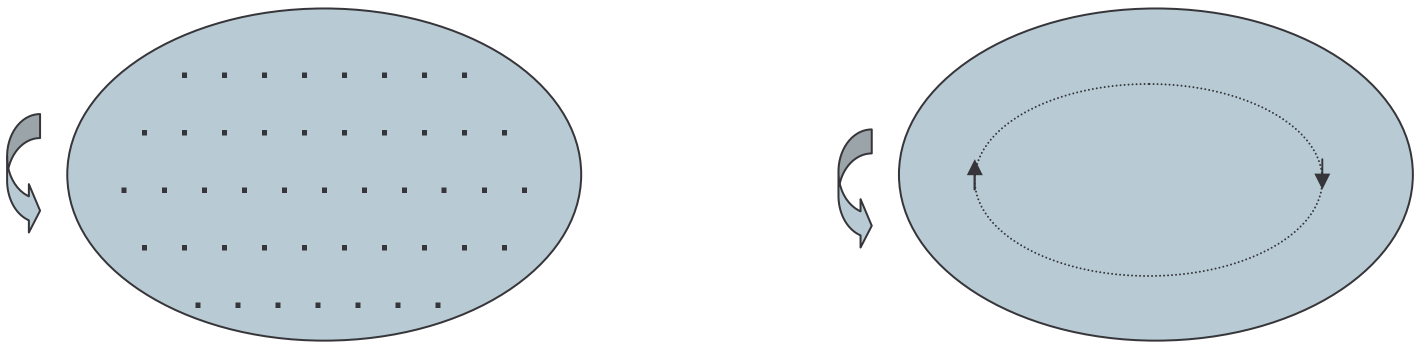
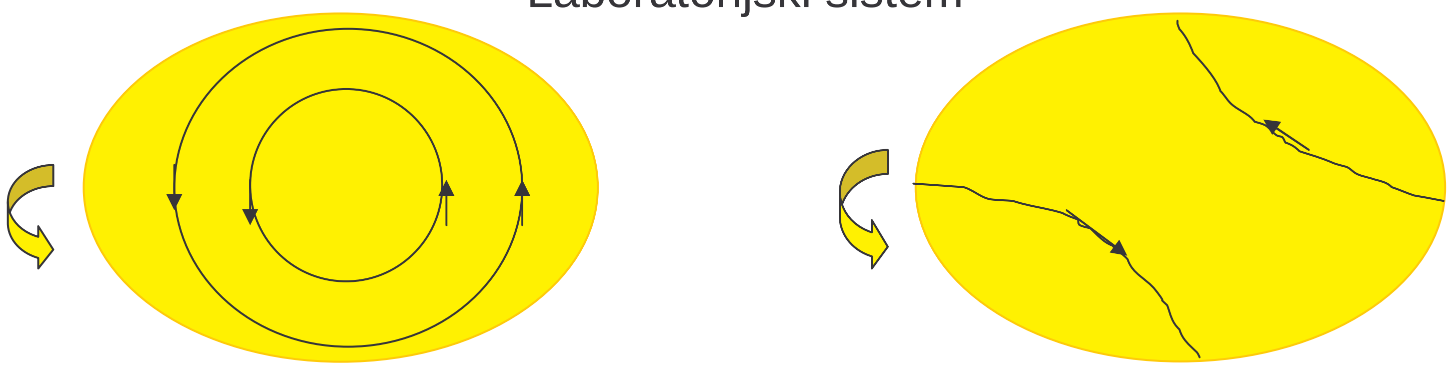
6. Surovo ali kuhano jajce?

Telesni sistem



Laboratorijski sistem



7. Jajce in vrtavka se postavi pokonci

JAJCE: zasučemo ga okrog prečne osi , ki je zaradi $I_1 = I_2$ nestabilna in se os vrtenja zaradi motenj podlage in trenja kmalu zruka in se in preide v os vrtenja okrog simetrijske osi, I_3 ◄ I_1, I_2 .

VRTAVKA: Poleg vrtenja se kotali po obli polkrogelni “konici” (“sklopitev spinske in tirne vrtilne količine”). Navor zaradi trenja sili vektor vrtilne količine vstran, dokler se ne prevrne v stabilnejšo lego na ostrejši konici. Seveda se mora vrteti dovolj hitro, da lahko plača pridobljeno potencialno energijo s kinetično energijo (pri čemer se ji nekoliko zmanjša kotna hitrost.

8. Vrtavka, ki spremeni smer

To je huda čarovnija, saj vrtavka (”Celtic stone”) navidez prekrši ohranitev vrtilne količine.

Vrtavka je navidez simetrična, v resnici pa ima vgrajeno nekje ob strani utež, ki ji pri eni smeri vrtenja pomaga – mirno jo vleče s seboj, pri drugi smeri vrtenja pa jo poriva in ruka. Navor zaradi trenja jo potem ne samo zagunca, temveč tudi spremeni smer vrtenja. Izpeljava je zelo komplicirana.

Vrtilna količina se seveda ni ohranila, saj so delovali navori!

Rattleback
From Wikipedia, the free encyclopedia
Jump to: navigation, search
For the fictional animal, see Rattleback (rodent).

Carved wooden rattleback
A **rattleback**, also known as an "anagyre", "celt", "Celtic stone", "rebellious celt", "rattlerock", "spin bar", "wobble stone" or "wobblestone" and by the product names "ARK," "Bizzaro Swirls," "RATTLEBACKS," "Space Pet" and "Space Toy," is a semi-ellipsoidal top which will spin on its axis in a preferred direction. But, if spun in the opposite direction, it becomes unstable, "rattles", stops and reverses its spin to the preferred direction.

Behold the mysterious celt,
with a property that amuses.
One way it will spin,
the other way it refuses.

This spin-reversal motion seems, at first sight, to violate the angular-momentum conservation law of physics. Moreover, for most rattlebacks, the motion will happen when the rattleback is spun in one direction, but not when spun in the other. Some exceptional rattlebacks will reverse when spun in either direction. [1] [2] This makes the rattleback a physical curiosity that has excited human imagination since prehistorical times.

The spin-reversal motion follows from the growth of instabilities on the other rotation axes, that are rolling (on the main axis) and pitching (on the crosswise axis).

Rolling and pitching motions

When there is an asymmetry in the mass distribution with respect to the plane formed by the pitching and the vertical axes, a coupling of these two instabilities arises; one can imagine how the asymmetry in mass will deviate the rattleback when pitching, which will create some rolling.

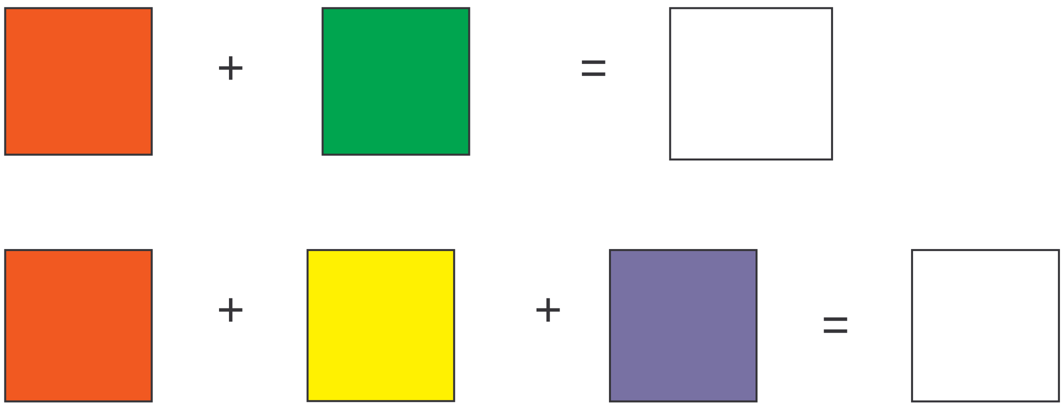
The amplified mode will differ depending on the spin direction, which explains the rattleback asymmetrical behavior. Depending on whether it is rather a pitching or rolling instability that dominates, the growth rate will be very high or quite low.

This explains why, due to friction, most rattlebacks exhibit spin-reversal motion only when spun in the pitching-unstable direction, while they slow down and stop spinning before the rolling instability arises when spun in the "stable" direction.

Also, after stopping, the spin in the "stable" direction is considerably slower than the original spin speed. Some rattlebacks, however, exhibit "unstable behavior" when spun in either direction, and incur several successive spin reversals per spin.

9. Vrtavka, ki spremeni barvo

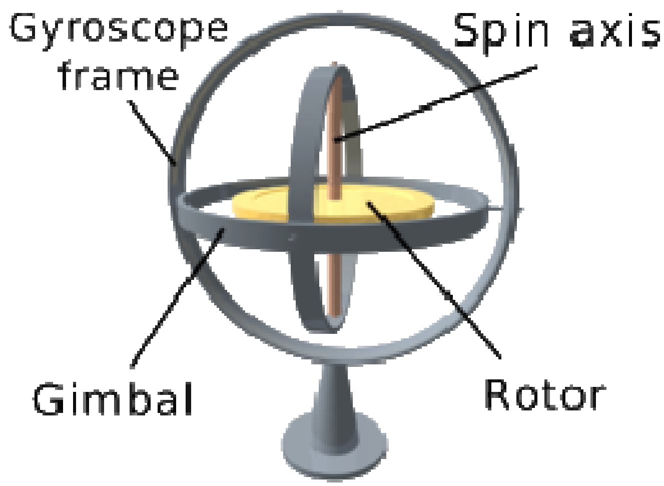
Znano je mešanje barv:

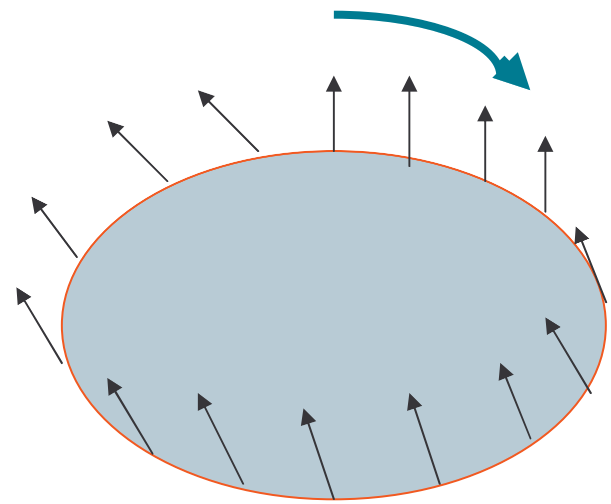


Preseneti pa, da da vrtavka z izmenoma belimi in črnimi progami namesto sive lahko rjavo ali rdečkasto ali modrikasto sivo barvo! ZAKAJ?

(Barvne čutnice v očesu reagirajo različno glede na trajanje in pogostost svetlobnega pulza)

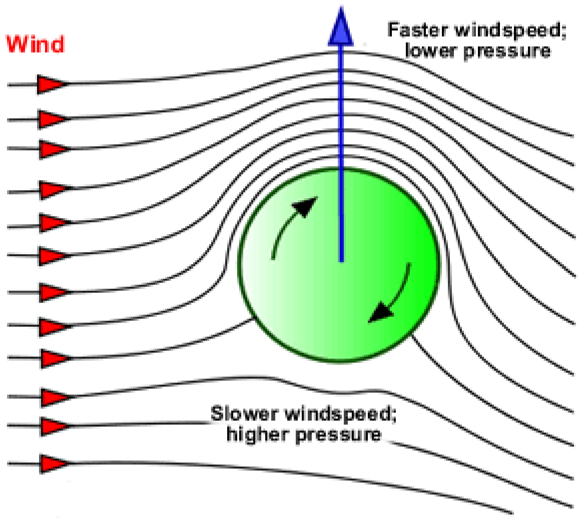
10. Giroskop. Vpliv zakrivljenega prostora

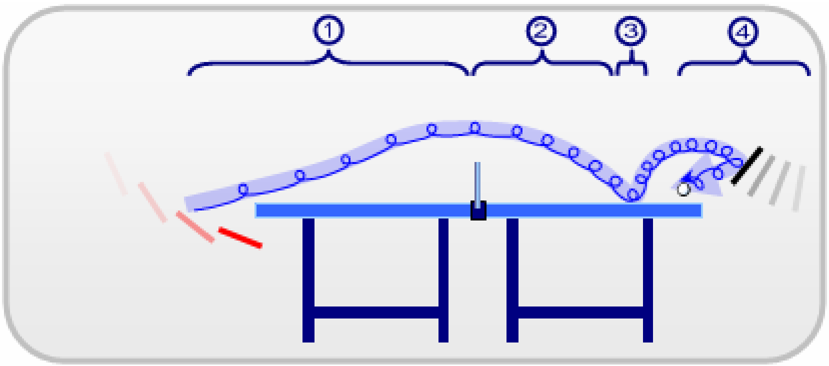




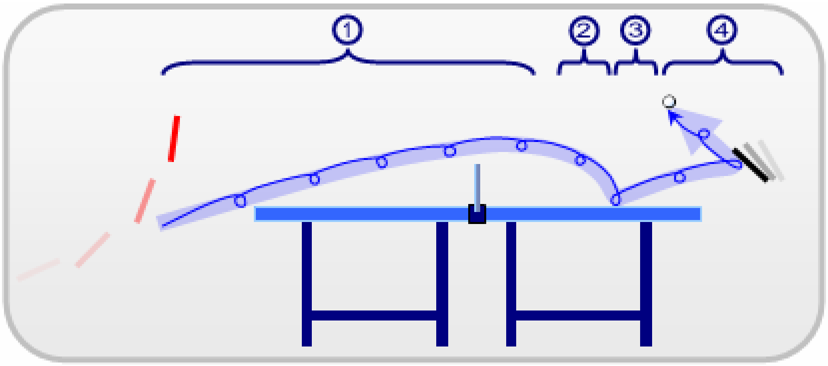
Če potuje giroskop po močnem graviracijskem polju, pride domov zaradi zakrivljenega prostora s spremenjenim spinom

11. Žoga (ping-pong, golf)





BACKSPIN



TOPSPIN

12. Bumerang

13. Jedrska magnetna resonanca

KLASIČNO: $\vec{\omega}_p \times \vec{L} = \vec{M} = \vec{B}_0 \times \vec{\mu}$

Ker sta kota na obeh straneh ista, $\vec{\omega}_p || \vec{B}_0$, $\vec{L} || \vec{\mu}$

smemo znak vektorskega produkta izpustiti:

$\omega_p \, L = B_0 \, \mu$.

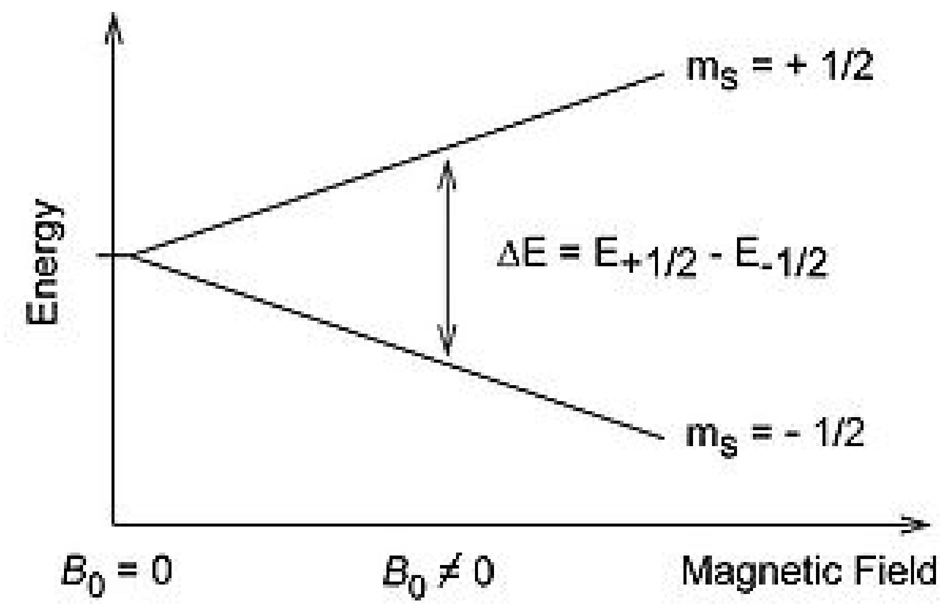
Vzemimo jedro s spinom $L = j\hbar$

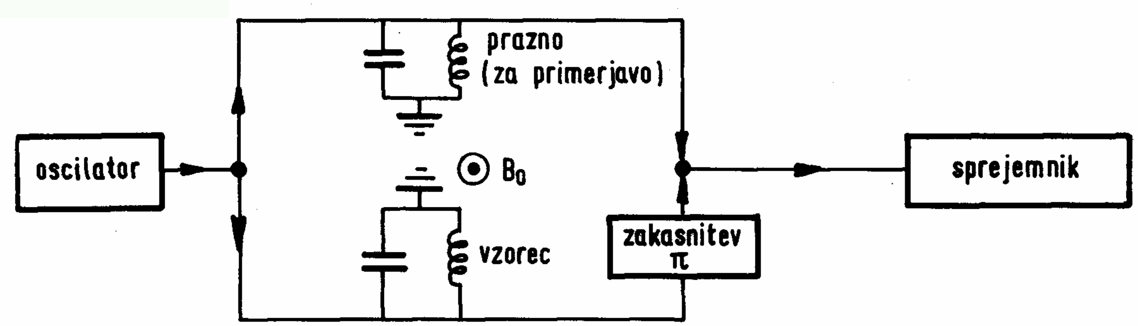
in magnetnim momentom $\mu = g(e\hbar/2m_p)$,

kjer je g giromagnetno razmerje.

$$\omega_p = \frac{B_0 \mu}{L} = \frac{g}{j} \frac{eB_0}{2m_p}$$

KVANTNO: Energijski nivoji magnetnega momenta v magnetnem polju so $E_m = \vec{\mu} \cdot \vec{B}_0 = \mu_z B_0$.
$\mu_z = (m/j) g (e\hbar/2m_p)$.
Pri preskoku med sosednjima nivojema ($\Delta m = 1$) odnese foton energijo
$\hbar\omega_p = \frac{g}{j} \frac{e\hbar B_0}{2m_p}$,
kar da isto ko klasično.





14. Ali se plesalka vrti v levo ali v desno?

Ali se plesalka vrti v levo ali v desno?

Mozgane sestavljata dve hemisferi, desna in leva. Vsaka polovica je odgovorna za svoje naloge, ena pa je vedno dominantna. Katera vodi vase misljenje?

Če je pri vas bolj dejavna desna polovica mozganov, se plesalka vrti **v desno** v smeri urnega kazalca, če pa se vrti **v levo,** je dominantnejsa leva polovica

Ljudje, pri katerih je dejavnejsa **desna polovica mozganov**, so bolj čustveni in manj pozorni na detajle. Uporabljajo domisljijo, se ukvarjajo z vero in filozofijo. Pripravljeni so tudi tvegati in imajo odlično prostorsko predstavo. Pogosto jih označujejo za sanjače in umetnike, ker imajo tezave z dejstvi in stevilkami. A to ne pomeni, da so neumni.

<u>Če je dejavnejsa leva polovica:</u>
To so ljudje, ki jih vodi logika. So zelo pozorni na detajle in pravila. So tudi dobri govorci in osredotočeni na sedanjost. Niso pripravljeni tvegati in so zelo praktične narave.

Zgodbe

- Kako mačku uspe, da vedno pade na noge?
- Kako je podmornica Nautilus leta 1958 navigirala 95 ur pod ledom na S tečaju?
- Zakaj se vodni vrtinec pri iztekanju iz plastenke ne ustavi?
- Zakaj so cevi pušk in topov znotraj narezane v spiralo?